# Reply

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We agree with Calder's comment that the solution for the area source problem can also be obtained by the superposition of the solutions for an infinite cross-wind line source. The solution can be found in several different ways, including the use of integral transforms. We hope that the method we have presented is particularly transparent and simple and feel it can be used to analyze other related diffusion problems. Since Robert's solution for the line source has not been published we are not aware of his method for solving the diffusion equation; however, an approach similar to ours was applied to the line source problem by Frost (1946).

The relation between the area source and the line source problems can be seen by comparing their emission boundary conditions. If $c_1(x,z)$ is the solution for the area source extending from $x=0$ to $x=\infty$, it satisfies the boundary condition

$$K(z)\frac{\partial c_1}{\partial z}=-Q\theta(x) \quad \text{at } z=0, \tag{1}$$

where $Q$ is the rate of emission from the surface and $\theta$ the step function. We note that $c_2(x,z)=\partial c_1/\partial x$ is also a solution of the diffusion equation. Differentiation of (1) with respect to $x$ gives the corresponding boundary condition for $c_2(x,z)$:

$$K(z)\frac{\partial c_2}{\partial z}=-Q\delta(x) \quad \text{at } z=0. \tag{2}$$

Eq. (2) can be recognized as the flux condition for a line source located at $x=z=0$. Thus $c_2(x,z)$ is the solution of the line source problem.

Eq. (2) describes a flux intensity which is zero everywhere except at $x=z=0$ where it approaches infinity In usual formulations of the line source problem (see e.g. Pasquill, 1974) this property is specified by prescribing $c(x,z)=\infty$ at $x=z=0$. Since the boundary conditions in the area source problem involve only finite quantities it appears to us that, in general, the procedure of first obtaining the area source solution and then deriving the line source solution by differentiation is to be preferred over the reverse route of an *ab initio* determination of the line source solution and obtaining the area source solution from it by superposition. The difference can be important when approximate solutions of the diffusion equation are sought

Although Robert's solution for the line source has been available for many years, the exact solution for the area source problem was not presented until now and a number of approximate methods have been used for modeling dispersion from area sources (see, e.g., references cited by Calder, 1970). Hopefully, the availability of the area source solution would result in improved modeling of air pollution dispersal over area sources

## REFERENCES

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Pasquill, F., 1974: *Atmospheric Diffusion*, 2nd ed. Wiley, 109 pp.